تصحيح الإمتحان الوطني
2024

التمرين 1 :

1) أ) تحقق أن : $\forall n\in\mathbb{N}:\ u_{n+1}=4-\frac{6}{1+u_n}$

$$u_{n+1}=\frac{4u_n-2}{1+u_n}$$
$$=\frac{4.u_n+4-4-2}{1+u_n}$$
$$=\frac{4(u_n+1)-6}{1+u_n}$$
$$=\frac{4(1+u_n)}{1+u_n}-\frac{6}{1+u_n}$$
$$=4-\frac{6}{1+u_n}.$$

ب) بين أن: $\forall n\in\mathbb{N}:\ 2\leq u_n\leq 4$

من أجل $n=0$

$2\leq u_0=4\leq 4$ عبارة صحيحة

ليكن $n\in\mathbb{N}$

نفترض أن $2\leq u_n\leq 4$

ونبين أن $2\leq u_{n+1}\leq 4$.

* لدينا: $2\leq u_n\leq 4$

$$\Rightarrow 3\leq 1+u_n\leq 5$$
$$\Rightarrow \frac{1}{5}\leq\frac{1}{1+u_n}\leq\frac{1}{3}$$
$$\Rightarrow -\frac{6}{3}\leq\frac{-6}{1+u_n}\leq-\frac{6}{5}$$
$$\Rightarrow -2\leq-\frac{6}{1+u_n}\leq-\frac{6}{5}$$
$$\Rightarrow 4-2\leq 4-\frac{6}{1+u_n}\leq 4-\frac{6}{5}$$
$$\Rightarrow 2\leq u_{n+1}\leq\frac{20-6}{5}$$

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$$\Rightarrow 2\leq u_{n+1}\leq\frac{14}{5}\leq 4$$

إذا $\Rightarrow 2\leq u_{n+1}\leq 4$

ومنه حسب مبدأ الترجع:

$$\forall n\in\mathbb{N}:\ 2\leq u_n\leq 4$$

2) أ) بين أن: $\forall n\in\mathbb{N}:\ u_{n+1}-u_n=\frac{(u_n-1)(2-u_n)}{1+u_n}$

* $$u_{n+1}-u_n=\frac{4u_n-2}{1+u_n}-u_n$$
$$=\frac{4u_n-2-u_n(1+u_n)}{1+u_n}$$
$$=\frac{4u_n-2-u_n-u_n^2}{1+u_n}$$
$$=\boxed{\frac{-u_n^2+3u_n-2}{1+u_n}}.$$

** $$\frac{(u_n-1)(2-u_n)}{1+u_n}=\frac{2u_n-u_n^2-2+u_n}{1+u_n}$$
$$=\boxed{\frac{-u_n^2+3u_n-2}{1+u_n}}.$$

C/C $$u_{n+1}-u_n=\frac{(u_n-1)(2-u_n)}{1+u_n}$$

ب) لنبين أن u_n تناقصية :

لدينا: $$u_{n+1}-u_n=\frac{(u_n-1)(2-u_n)}{1+u_n}$$

$$2\leq u_n\leq 4\Rightarrow\boxed{1\leq u_n-1\leq 3}$$
$$2\leq u_n\leq 4\Rightarrow -4\leq -u_n\leq -2$$
$$\Rightarrow\boxed{-2\leq 2-u_n\leq 0}$$
$$2\leq u_n\leq 4\Rightarrow\boxed{3\leq 1+u_n\leq 5}$$

متقاربة:

لدينا U_n تناقصية ومصغورة بـ 2

اذن U_n متقاربة:

3) أ) لنبين أن V_n هندسية أساسها $\frac{2}{3}$.

$$V_{n+1} = \frac{2-U_{n+1}}{1-U_{n+1}}$$

$$= \frac{2-\frac{4U_n-2}{1+U_n}}{1-\frac{4U_n-2}{1+U_n}}$$

$$= \frac{\frac{2(1+U_n)-(4U_n-2)}{\cancel{1+U_n}}}{\frac{1+U_n-(4U_n-2)}{\cancel{1+U_n}}}$$

$$= \frac{2+2U_n-4U_n+2}{1+U_n-4U_n+2}$$

$$= \frac{-2U_n+4}{-3U_n+3}$$

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$$= \frac{2(-U_n+2)}{3(-U_n+1)}$$

$$= \frac{2}{3}\,\frac{2-U_n}{1-U_n}$$

$$= \frac{2}{3}\,V_n.$$

اذن V_n متتالية هندسية أساسها $q=\frac{2}{3}$.

ب) بين أن:

$$\forall n\in\mathbb{N},\ U_n = 1+\frac{1}{1-\left(\frac{2}{3}\right)^{n+1}}.$$

لدينا: V_n هندسية اذن:

$$V_n = V_p\cdot q^{n-p}$$

$$V_n = V_0\cdot q^n = \frac{2}{3}\cdot\left(\frac{2}{3}\right)^n = \left(\frac{2}{3}\right)^{n+1}$$

$$V_0 = \frac{2-U_0}{1-U_0} = \frac{2-4}{1-4} = \frac{-2}{-3} = \frac{2}{3}.$$ لأن

$$* \quad V_n = \frac{2-U_n}{1-U_n}$$

$$\Rightarrow V_n(1-U_n) = 2-U_n$$

$$\Rightarrow V_n - V_n\cdot U_n = 2-U_n.$$

$$\Rightarrow U_n - V_n\cdot U_n = 2-V_n$$

$$\Rightarrow U_n(1-V_n) = 2-V_n$$

$$\Rightarrow U_n = \frac{2-V_n}{1-V_n}$$

$$\Rightarrow U_n = \frac{1-V_n}{1-V_n}+\frac{1}{1-V_n}$$

$$\Rightarrow U_n = 1+\frac{1}{1-V_n}$$

$$\Rightarrow \boxed{U_n = 1+\frac{1}{1-\left(\frac{2}{3}\right)^{n+1}}}$$

2) $\lim\limits_{n\to+\infty} U_n.$

$$\lim_{+\infty} U_n = \lim_{+\infty} 1+\frac{1}{1-\left(\frac{2}{3}\right)^{n+1}}$$

$$= 1+1 = 2$$

لأن

$$\lim_{+\infty}\left(\frac{2}{3}\right)^{n+1} = 0\ ,\ -1<\frac{2}{3}<1$$

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التمرين 2 :

1) بين أن $(P): 2x - 2y + z + 3 = 0.$

المستوى (P) مار من A و $\vec{n}$ منظمية عليه إذن:

$$(P): 2x - 2y + z + d = 0.$$

وبما أن $A(-1;0;-1) \in (P)$

إذن:

$$2x_A - 2y_A + z_A + d = 0$$

$$2\times(-1) - 2\times 0 + (-1) + d = 0$$

$$-2 - 0 - 1 + d = 0$$

$$-3 + d = 0$$

$$\boxed{d = 3}$$

إذن $\boxed{(P): 2x - 2y + z + 3 = 0}$

2) المعادلة الديكارتية للفلكة (S):

$\Omega(2;-1;0)$ و $R = 5.$

$$(x-2)^2 + (y-(-1))^2 + (z-0)^2 = 5^2$$

$$(x-2)^2 + (y+1)^2 + z^2 = 25$$

$$x^2 - 4x + 4 + y^2 + 2y + 1 + z^2 = 25$$

$$x^2 + y^2 + z^2 - 4x + 2y + 5 - 25 = 0$$

$$\boxed{x^2 + y^2 + z^2 - 4x + 2y - 20 = 0}$$

3) أ)

$$d(\Omega;(P)) = \frac{|2x_\Omega - 2y_\Omega + z_\Omega + 3|}{\sqrt{2^2 + (-2)^2 + 1^2}}$$

$$= \frac{|2\times 2 - 2\times(-1) + 0 + 3|}{\sqrt{4+4+1}}$$

$$= \frac{|4+2+3|}{\sqrt{9}} = \frac{9}{3} = 3$$

ب) لدينا $d(\Omega;(P)) = 3 < R$

إذن المستوى (P) يقطع الفلكة (S) وفق دائرة (Γ) مركزها:

$$r = \sqrt{R^2 - d^2}$$

$$= \sqrt{5^2 - 3^2}$$

$$= \sqrt{25 - 9}$$

$$= \sqrt{16}$$

$$\boxed{r = 4}$$

4) أ) لدينا (Δ) المستقيم المار من Ω وعمودي على المستوى (P) إذن $\vec{n}$ متجهة موجهة للمستقيم (Δ):

$$(\Delta): \begin{cases} x = x_\Omega + at \\ y = y_\Omega + bt \\ z = z_\Omega + ct \end{cases} \quad / t \in \mathbb{R}$$

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$$(\Delta): \begin{cases} x = 2 + 2t \\ y = -1 - 2t \\ z = t \end{cases} \quad / t \in \mathbb{R}$$

ب) لدينا (Δ) مار من Ω وعمودي على (P) والمستوى (P) يقطع الفلكة (S) وفق دائرة (Γ) إذن الإسقاط العمودي لـ Ω على (P) هو H

$$H = (P) \cap (\Delta).$$

$$\begin{cases} 2x - 2y + z + 3 = 0 \\ \begin{cases} x = 2 + 2t \\ y = -1 - 2t \\ z = t \end{cases} / t \in \mathbb{R} \end{cases}$$

$$\Rightarrow 2(2+2t) - 2(-1-2t) + t + 3 = 0$$

$$\Rightarrow 4 + 4t + 2 + 4t + t + 3 = 0$$

$$\Rightarrow 9t + 9 = 0$$

$$\Rightarrow 9t = -9 \Rightarrow \boxed{t = -1}$$

$$\arg(a) \equiv -\frac{\pi}{4} \quad [2\pi].$$

ب (2

$$\frac{b}{a} = \frac{2+\sqrt{3}+i}{\sqrt{3}(1-i)}$$

$$= \frac{2+\sqrt{3}+i}{\sqrt{3}-\sqrt{3}i}$$

$$= \frac{(2+\sqrt{3}+i)(\sqrt{3}+\sqrt{3}i)}{(\sqrt{3}-\sqrt{3}i)(\sqrt{3}+\sqrt{3}i)}$$

$$= \frac{(2+\sqrt{3}).\sqrt{3}+(2+\sqrt{3}).\sqrt{3}i+i\sqrt{3}+i^2.\sqrt{3}}{\sqrt{3}^2-(i\sqrt{3})^2}$$

$$= \frac{2\sqrt{3}+3+2\sqrt{3}i+3i+i\sqrt{3}-\sqrt{3}}{3+3}$$

$$= \frac{3+\sqrt{3}+3\sqrt{3}i+3i}{6}$$

4

$$= \frac{3+\sqrt{3}}{6} + \frac{3(\sqrt{3}+1)i}{6}$$

$$\frac{b}{a} = \frac{3+\sqrt{3}}{6} + \frac{\sqrt{3}+1}{2}i \quad \boxtimes$$

$$\frac{b}{a} = \frac{3+\sqrt{3}}{6} + \frac{\sqrt{3}+1}{2}i$$

$$= \frac{3+\sqrt{3}}{3}\left(\frac{1}{2} + \frac{\sqrt{3}+1}{2} \times \frac{3}{3+\sqrt{3}}i\right)$$

$$= \frac{3+\sqrt{3}}{3}\left(\frac{1}{2} + \frac{\cancel{\sqrt{3}+1}}{2} \times \frac{3}{\cancel{\sqrt{3}}(\cancel{1+\sqrt{3}})}i\right)$$

$$= \frac{3+\sqrt{3}}{3}\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$$

$$= \frac{3+\sqrt{3}}{3}.\left(\cos\frac{\pi}{3} + i.\sin\frac{\pi}{3}\right)$$

$$= \frac{3+\sqrt{3}}{3}.e^{i.\frac{\pi}{3}}.$$

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$$\begin{cases} x_H = 2+2\times(-1) = 0 \\ y_H = -1-2\times(-1) = 1 \\ z_H = -1 \end{cases}$$

$$\boxed{H(0\,;1\,;-1).}$$

ج) لنبين أن (Δ) واسط القطعة $[AB]$.

$\vec{n}(2;-2;1) \qquad \vec{AB}(2;2;0)$

$$\vec{n}.\vec{AB} = 2\times2+(-2)\times2+1\times0$$
$$= 4-4$$
$$= 0$$

إذن $(AB) \perp (\Delta)$.

لنحدد احداثيات منتصف القطعة $[AB]$

$$\frac{x_A+x_B}{2} = \frac{1+(-1)}{2} = 0$$

$$\frac{y_A+y_B}{2} = \frac{2+0}{2} = \frac{2}{2} = 1.$$

$$\frac{z_A+z_B}{2} = \frac{(-1)+(-1)}{2} = \frac{-2}{2} = -1.$$

إذن H منتصف القطعة $[AB]$.

و $H \in (\Delta)$

إذن (Δ) هو واسط القطعة $[AB]$

التمرين 3:

1)

$$a = \sqrt{3}(1-i)$$

$$|a| = |\sqrt{3}(1-i)|$$
$$= |\sqrt{3}|.|1-i|$$
$$= \sqrt{3}.\sqrt{1^2+(-1)^2}$$
$$= \sqrt{3}.\sqrt{2}$$
$$= \sqrt{6}$$

$$a = \sqrt{3}(1-i)$$
$$= \sqrt{6}\left(\frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}\right)$$
$$= \sqrt{6}\left(\cos\left(-\frac{\pi}{4}\right) + i.\sin\left(-\frac{\pi}{4}\right)\right)$$

3) أ

$$z' - w = e^{i\theta}(z - w)$$

$$z' - 0 = e^{i\pi/6} \cdot (z - 0)$$

$$\Rightarrow z' = e^{i\pi/6} \cdot z$$

$$\Rightarrow z' = \left(\cos\frac{\pi}{6} + i \cdot \sin\frac{\pi}{6}\right) \cdot z$$

$$\Rightarrow z' = \left(\frac{\sqrt{3}}{2} + i\,\frac{1}{2}\right) \cdot z$$

$$\Rightarrow \boxed{z' = \frac{1}{2}(\sqrt{3} + i) \cdot z}$$

$$R(A) = A'$$

$$\Rightarrow a' = \frac{1}{2}(\sqrt{3} + i) \cdot a.$$

$$\arg(a') \equiv \arg\left(\frac{1}{2}(\sqrt{3}+i) \cdot a\right) \quad [2\pi]$$

$$\equiv \arg\left(\frac{1}{2}(\sqrt{3}+i)\right) + \arg(a) \quad [2\pi]$$

$$\equiv \frac{\pi}{6} - \frac{\pi}{4} \quad [2\pi]$$

$$\equiv \frac{2\pi - 3\pi}{12} \quad [2\pi]$$

$$\boxed{\arg(a') \equiv -\frac{\pi}{12} \quad [2\pi].}$$

$$R(A') = A''$$

$$\Rightarrow a'' = \frac{1}{2}(\sqrt{3} + i) \cdot a'$$

$$\Rightarrow a'' = \left(\frac{1}{2}(\sqrt{3}+i)\right)^2 \cdot a.$$

$$|a''| = \left|\frac{\sqrt{3}}{2} + \frac{1}{2}i\right|^2 \cdot |a|$$

$$= 1 \cdot \sqrt{6}$$

$$= \boxed{\sqrt{6}}.$$

$$\arg(a'') \equiv \arg\left(\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)^2 \cdot a\right) \quad [2\pi]$$

$$\equiv \arg\left(\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)^2\right) + \arg(a) \quad [2\pi]$$

$$\equiv 2 \cdot \arg\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right) + \arg(a) \quad [2\pi]$$

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2) ب لدينا

$$\frac{b}{a} = \frac{3+\sqrt{3}}{3} \cdot e^{i\frac{\pi}{3}}$$

$$\Rightarrow b = \left(\frac{3+\sqrt{3}}{3} \cdot e^{i\pi/3}\right) \cdot a.$$

$$|b| = \left|\frac{3+\sqrt{3}}{3} \cdot e^{i\pi/3} \cdot a\right|$$

$$= \left|\frac{3+\sqrt{3}}{3} \cdot e^{i\pi/3}\right| \cdot |a|$$

$$= \frac{3+\sqrt{3}}{3} \cdot \sqrt{6}.$$

$$= \frac{(3+\sqrt{3}) \cdot \sqrt{2}}{\sqrt{3}} = \boxed{\frac{3\sqrt{2} + \sqrt{6}}{\sqrt{3}}} = \boxed{\sqrt{2}+\sqrt{6}}$$

$$\arg(b) \equiv \arg\left(\frac{3+\sqrt{3}}{3} \cdot e^{i\pi/3} \cdot a\right) \quad [2\pi]$$

$$\equiv \arg\left(\frac{3+\sqrt{3}}{3} \cdot e^{i\pi/3}\right) + \arg(a) \quad [2\pi]$$

$$\equiv \frac{\pi}{3} - \frac{\pi}{4} \quad [2\pi]$$

$$\boxed{\arg(b) \equiv \frac{\pi}{12} \quad [2\pi]}$$

ج

$$\boxed{b = \frac{3\sqrt{2}+\sqrt{6}}{\sqrt{3}} \cdot \left(\cos\frac{\pi}{12} + i \cdot \sin\frac{\pi}{12}\right)}$$

$$= (\sqrt{2}+\sqrt{6})\left(\cos\pi/12 + i \cdot \sin\pi/12\right) = (\sqrt{2}+\sqrt{6})e^{i\pi/12}$$

$$b^{24} = \left(\frac{3\sqrt{2}+\sqrt{6}}{\sqrt{3}} \cdot \left(\cos\frac{\pi}{12} + i \cdot \sin\frac{\pi}{12}\right)\right)^{24}$$

$$= \left(\frac{3\sqrt{2}+\sqrt{6}}{\sqrt{3}}\right)^{24} \cdot \left(\cos\left(24\frac{\pi}{12}\right) + i \cdot \sin\left(24\frac{\pi}{12}\right)\right)$$

$$= \left(\frac{3\sqrt{2}+\sqrt{6}}{\sqrt{3}}\right)^{24} \cdot \left(\underbrace{\cos(2\pi)}_{1} + i \cdot \underbrace{\sin(2\pi)}_{0}\right)$$

$$b^{24} = \left(\frac{3\sqrt{2}+\sqrt{6}}{\sqrt{3}}\right)^{24}$$

$$= \left(\frac{3\sqrt{6}+3\sqrt{2}}{3}\right)^{24} = (\sqrt{6}+\sqrt{2})^{24} \in \mathbb{R}$$

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$$\boxed{b' = \frac{3+\sqrt{3}}{3} \cdot \overline{a}}.$$

2)

$$\arg\left(\frac{b'-0}{a-0}\right) \equiv \arg\left(\frac{b'}{a}\right) [2\pi]$$

$$\equiv \arg\left(\frac{\frac{3+\sqrt{3}}{3} \cdot \overline{a}}{a}\right) [2\pi]$$

$$\equiv \arg\left(\frac{\frac{3+\sqrt{3}}{3} \cdot i \cdot \cancel{a}}{\cancel{a}}\right) [2\pi]$$

$$\equiv \arg\left(\frac{3+\sqrt{3}}{3} \cdot i\right) [2\pi]$$

$$\equiv \arg(i)\ [2\pi] \quad \left(\frac{3+\sqrt{3}}{3} \in \mathbb{R}^+\right) \text{ لأن:}$$

$$\equiv \frac{\pi}{2}\ [2\pi]$$

اذن المثلث OAB' قائم الزاوية في O.

التمرين 4 :

1 1	1 1	2 2	3

1) 11 أو 22

$$P(A) = \frac{C_4^2 + C_2^2}{C_7^2} = \frac{6+1}{21} = \frac{7}{21} = \frac{1}{3}.$$

2) 13 أو 22

$$P(B) = \frac{C_4^1 \cdot C_1^1 + C_2^2}{C_7^2} = \frac{4\times1+1}{21} = \frac{5}{21}$$

3) $A\cap B = 22$

$$P(A\cap B) = \frac{C_2^2}{C_7^2} = \frac{1}{21}.$$

4)

$$P(A)\times P(B) = \frac{1}{3}\times\frac{5}{21} = \frac{5}{63} \neq \frac{1}{21} = P(A\cap B)$$

اذن الحدثان A و B غير مستقلان.

لأن $P(A)\times P(B) \neq P(A\cap B)$

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$$\arg(a'') \equiv \frac{2\pi}{6} + \left(-\frac{\pi}{4}\right) [2\pi]$$

$$\equiv \frac{\pi}{3} - \frac{\pi}{4}\ [2\pi]$$

$$\equiv \boxed{\frac{\pi}{12}\ [2\pi]}$$

$$a'' = \sqrt{6}\cdot\left(\cos\frac{\pi}{12} + i\cdot\sin\frac{\pi}{12}\right)$$

$$\boxed{a'' = \sqrt{6}\cdot e^{i\cdot\pi/12}}$$

6)

$$\frac{b-0}{a''-0} = \frac{(\sqrt{2}+\sqrt{6})\cdot(\cos\pi/12 + i\cdot\sin\pi/12)}{\sqrt{6}\cdot e^{i\pi/12}}$$

$$= \frac{(\sqrt{2}+\sqrt{6})\cdot \cancel{e^{i\pi/12}}}{\sqrt{6}\cdot\cancel{e^{i\pi/12}}}$$

$$= \frac{\sqrt{2}+\sqrt{6}}{\sqrt{6}} = \frac{\sqrt{12}+6}{6}$$

$$= \frac{\sqrt{3}+3}{3} \in \mathbb{R}$$

اذن O, A'' و B نقط مستقيمية.

7)

$$R(B) = B'$$

$$\Rightarrow b' = \frac{1}{2}(\sqrt{3}+i)\cdot b$$

$$= \left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)\cdot \frac{3+\sqrt{3}}{3}\cdot e^{i\pi/3}\cdot a.$$

$$b' = \frac{3+\sqrt{3}}{3}\cdot\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)\cdot\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)\cdot a$$

$$= \frac{3+\sqrt{3}}{3}\left(\cancel{\frac{\sqrt{3}}{4}} + i\frac{3}{4} + \frac{1}{4}i - \cancel{\frac{\sqrt{3}}{4}}\right)\cdot a$$

$$= \frac{3+\sqrt{3}}{3}\cdot i\cdot a$$

$$= \frac{3+\sqrt{3}}{3}\, i\left(\sqrt{3}(1-i)\right)$$

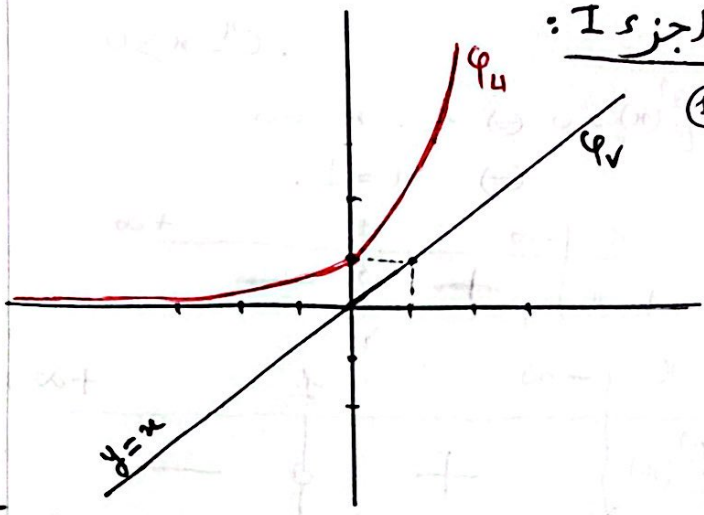
$$= \frac{3+\sqrt{3}}{3}\sqrt{3}\left(i(1-i)\right)$$

$$= \frac{3+\sqrt{3}}{3}\cdot\sqrt{3}\cdot(i+1)$$

المسألة :

الجزء I :

1)



2) لدينا (φ_v) تحت (φ_u) في الشكل السابق

إذن $u(x) > v(x)$.

ومنه $u(x) - v(x) > 0$

$e^x - x > 0$.

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3)

$$\mathcal{A} = \int_0^1 |u(x) - v(x)| \, dx \; . \; u.a.$$

$$= \int_0^1 |e^x - x| \, dx \; . \; u.a$$

$$= \int_0^1 (e^x - x) \, dx \; . \; u.a \qquad \left(\text{لأن } e^x - x > 0\right)$$

$$= \left[e^x - \frac{x^2}{2}\right]_0^1 . \; u.a.$$

$$= \left[e - \frac{1}{2} - (e^0 - 0)\right] . \; u.a.$$

$$= \left(e - \frac{1}{2} - 1\right) . \; u.a.$$

$$= e - \frac{3}{2} \; . \; u.a.$$

الجزء II :

$$f(x) = x + 1 - \ln(e^x - x).$$

1) أ)

$$D_f = \{x \in \mathbb{R} \, / \, e^x - x > 0\}.$$
$$= \mathbb{R}$$

حسب السؤال 2) I .

ب)

$$f(x) = x + 1 - \ln(e^x - x)$$
$$= x + 1 - \ln\left(e^x(1 - x.e^{-x})\right)$$
$$= x + 1 - \ln(e^x) - \ln(1 - x.e^{-x})$$
$$= \cancel{x} + 1 - \cancel{x} - \ln(1 - x.e^{-x})$$
$$= \boxed{1 - \ln(1 - x.e^{-x})}.$$

ج)

$$\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} 1 - \ln(1 - x.e^{-x})$$
$$= \lim_{+\infty} 1 - \ln\left(1 - \frac{x}{e^x}\right)$$
$$= \lim_{+\infty} 1 - \ln\left(1 - \frac{1}{\underbrace{\frac{e^x}{x}}_{\to +\infty}}\right)$$
$$= 1 - \ln(1)$$
$$= 1 \qquad \left(\text{car } \lim_{+\infty} \frac{e^x}{x} = +\infty\right)$$

(φ_f) يقبل مقارب أفقي معادلته $y = 1$ بجوار $+\infty$.

2) أ)

$$\lim_{-\infty} f(x) = \lim_{-\infty} x + 1 - \ln(e^x - x)$$
$$= -\infty$$

لأن :

$$\lim_{-\infty} x + 1 = -\infty$$
$$\lim_{-\infty} e^x - x = +\infty$$
$$\lim_{-\infty} \ln(e^x - x) = +\infty.$$

ب)

$$f(x) = x + 1 - \ln(e^x - x)$$
$$= x + 1 - \ln\left(-x\left(\frac{e^x}{-x} + 1\right)\right) \qquad \left(\text{لأن } x < 0 \Rightarrow -x > 0\right)$$
$$= x + 1 - \ln(-x) - \ln\left(1 + \frac{e^x}{-x}\right)$$

$$\boxed{f(x) = x + 1 - \ln(-x) - \ln\left(1 - \frac{1}{x e^{-x}}\right)}$$

ب) إشارة f' هي إشارة $1-x$ لأن $e^x - x > 0$.

$f'(x) = 0 \iff 1 - x = 0$
$\iff x = 1.$

x	$-\infty$		1		$+\infty$
$1-x$		$+$	0	$-$	

x	$-\infty$		1		$+\infty$
$f'(x)$		$+$	0	$-$	
$f(x)$	$-\infty$	↗	$2-\ln(e-1)$	↘	1

$f(1) = 1+1-\ln(e^1-1) = 2-\ln(e-1)$

ج) لدينا f متصلة على $\mathbb{R}$ (لأنها ق ش على $\mathbb{R}$) وبالخصوص على $[-1;0]$.
و f تزايدية قطعا على $[-1;0]$ (أنظر جدول التغيرات)

$f(-1) = -1+1-\ln(e^{-1}+1)$
$= -\ln\left(\frac{1}{e}+1\right)$

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$\frac{1}{e}+1 > 1 \Rightarrow \ln\left(\frac{1}{e}+1\right) > 0$
$\Rightarrow \boxed{-\ln\left(\frac{1}{e}+1\right) < 0}$

$f(0) = 0+1-\ln(e^0-0)$
$= 1-\ln(1)$
$= 1 > 0$

اذن
$f(-1)\times f(0) < 0$

ومنه حسب مبرهنة القيم الوسيطية T.V.I المعادلة $f(x)=0$ تقبل حلا وحيدا α في المجال $]-1;0[$.

2) ج)

$\lim_{-\infty} \frac{f(x)}{x} = \lim_{-\infty} \frac{x+1-\ln(-x)-\ln\left(1-\frac{1}{xe^{-x}}\right)}{x}$

$= \lim_{-\infty} \frac{x}{x} + \frac{1}{x} - \frac{\ln(-x)}{x} - \frac{\ln\left(1-\frac{1}{xe^{-x}}\right)}{x}$

$= \lim_{-\infty} 1 + \frac{1}{x} + \frac{\ln(-x)}{-x} - \frac{\ln\left(1+\frac{1}{-xe^{-x}}\right)}{x}$

$= \boxed{1}$

لأن:
$\lim_{-\infty} \frac{1}{x} = 0 \; ; \; \lim_{-\infty} \frac{\ln(-x)}{-x} = 0$

$\lim_{-\infty} \frac{\ln\left(1+\frac{1}{-x.e^{-x}}\right)}{x} = 0.$

$\lim_{-\infty} -x.e^{-x} = +\infty.$

* $\lim_{-\infty} f(x) - x = \lim_{-\infty} 1 - \ln(-x) - \ln\left(1-\frac{1}{xe^{-x}}\right)$

$= \lim_{-\infty} 1 - \underbrace{\ln(-x)}_{+\infty} - \underbrace{\ln\left(1+\frac{1}{-x.e^{-x}}\right)}_{0}.$

$= -\infty$

اذن $(\mathcal{C}_f)$ يقبل فرعا شلجميا في اتجاه المستقيم الذي معادلته $y=x$ بجوار $-\infty$.

3) أ)

$f'(x) = \left(x+1-\ln(e^x-x)\right)'$

$= 1+0-\frac{(e^x-x)'}{e^x-x}$

$= 1-\frac{e^x-1}{e^x-x}$

$= \frac{\cancel{e^x}-x-\cancel{e^x}+1}{e^x-x}$

$\boxed{f'(x) = \frac{1-x}{e^x-x}}$

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4) أ) نلاحظ من خلال الشكل أن (C_f) يقطع المستقيم $y=x$ في نقطتين أفصولهما α و β.

ب)

$$f(\alpha)=\alpha \Rightarrow f(\alpha)-\alpha=0$$

$$\Rightarrow 1-\ln(e^{\alpha}-\alpha)=0$$

$$\Rightarrow -\ln(e^{\alpha}-\alpha)=-1$$

$$\Rightarrow \ln(e^{\alpha}-\alpha)=1$$

$$\Rightarrow \boxed{e^{\alpha}-\alpha=e} \quad ①$$

$$f(\beta)=\beta \Rightarrow f(\beta)-\beta=0$$

$$\Rightarrow 1-\ln(e^{\beta}-\beta)=0$$

$$\Rightarrow \ln(e^{\beta}-\beta)=1.$$

$$\Rightarrow \boxed{e^{\beta}-\beta=e} \quad ②$$

$$① \text{ et } ② \Rightarrow e^{\alpha}-\alpha=e^{\beta}-\beta$$

$$\Rightarrow \boxed{e^{\alpha}-e^{\beta}=\alpha-\beta}.$$

5) أ) لدينا g متصلة على $]-\infty;1]$
(لأنها قابلة للاشتقاق على $]-\infty;1]$)
و g تزايدية قطعا على $]-\infty;1]$
(جدول التغيرات)
اذن g تقبل دالة عكسية g^{-1} معرفة على مجال J:

$$J=g(I)=g(]-\infty;1])$$

$$=]\lim_{-\infty} g(x);\ g(1)]$$

$$=]-\infty;2-\ln(e-1)].$$

ب) لدينا $f(0)=1$
اذن $g(0)=1.$

بما أن g قابلة للاشتقاق في 0
وأن $g'(0)=\frac{1-0}{e^0-0}=\frac{1}{1}=1 \neq 0$

اذن g^{-1} قابلة للاشتقاق في 1.

$$(g^{-1})'(1)=\frac{1}{g'(g^{-1}(1))}$$

$$=\frac{1}{g'(0)}$$

$$=\frac{1}{1}$$

$$(g^{-1})'(1)=1.$$

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Fin.